Basics 5:
Math Review

A Mathematical Interlude

Before we move on to a discussion of solutions, we want to review a few mathematics concepts. Some of these have been introduced earlier, and some will be immediately useful in our study of solutions. Although this is not all the math you ever need to know (you can never know enough math), it should get you through most of basic chemistry--it is a good idea to try and master it once and for all, if you have not don so already.

Ratios - what is a ratio?

The idea of the ratio is probably the most fundamental and ubiquitous idea in chemistry, and in most of the everyday math we do in the course of our lives. When you decide that 3 for a dollar is a better deal than $2.50 for a half dozen, you are using ratios. So we start our math interlude with the question: what is a ratio?

At its simplest - it is division --- but what is division? If multiplication is shorthand addition, is division shorthand subtraction??
It is a way of comparing similar things. We can compare relative salaries, height, weight or whatever between 2 people by finding their ratios.
It is a way of finding how much of one thing is necessary for another thing--this sounds very confusing, I know...but wait...it will come to you soon.

Let's look at the first idea--we know that multiplication is shorthand for addition: 10X12 = 120 is the same as saying if you add up 12 ten times you will get 120. Let's see if the same logic applies to division:

$$\begin{array}{l} \qquad 7 \quad \textit{and 4 left over} \\ 7\,\sqrt{53} \end{array}$$

Now try successive subtractions

$$\begin{array}{rrr} 53 & 32 & 11\\ -7 & -7 & -7\\ \hline 46 & 25 & 4\\ -7 & -7 & \\ \cline{1-2} 39 & 18 & \\ -7 & -7 & \\ \cline{1-2} 32 & 11 & \end{array}$$

See? We subtracted 7 times & had 4 left over. So a ratio is , in fact, a shorthand for subtraction. But it is more.

It is a method for comparison. If you are 6 feet tall (72 inches) and I am 5 foot 8 (68 inches) then the ratio:

$$\frac{72\ \cancel{\text{inches}}}{68\ \cancel{\text{inches}}} = \frac{18}{17}$$

expesses the relative size of our heights

(*note how the units cancel out*!)

But suppose the units don't cancel out and we are comparing dissimilar units. This is the third case that we said sounded so confusing above.

Let's look at it using a simple example. Suppose you buy 6 bagels for 1.5 dollars. Look at the ratio that results:

$$\frac{1.5\ \$}{6\ bagels} = 0.25\,{}^{\$}/_{bagels}\ represents$$

$$the\ num\ \text{ber}\ \ of\ dollars\ (\tfrac{1}{4})\ for\ each$$

$$bagel$$

You can turn the problem upside down

$$The\ ratio$$

$$\frac{6\ bagel}{1.5\ dollars} = \frac{4\ bagel}{dollar}$$

$$4\ bagels\ \bigcirc\!\!\!\!\!\!\!\!\!\!\ per\ \ dollar$$

$$ick \nearrow$$

See? The ratio expresses an inherent relationship between the number of bagels you buy and the price you must pay for them, just as the speed on your speedometer represents the relationship between how far you travel and the time it takes you to cover that distance.

Table of Fractions and Percentages

And now a word from our sponsor: you really ought to try to practice doing arithmetic in your head. It keeps you mental muscles strong. I recommend the following table as a good starting point.

I don't believe in memorization but I do believe in some remembering. I know this sounds contradictory...so sue me for it. All I know is that remembering the table below has gotten me through a whole lot of mathematical and arithmetical problems. It lets me "see through" things sometimes because

Frequently there are insights buried in the relationships between numbers. They give me lots of shortcuts which helps me see whether the answer I got makes sense.

I know...you have a calculator...and I use them too...but it surely is useful having some of this stuff right up there in your noodle where you can access it any time you want.

fraction	decimal	%		fraction	decimal	%
9/10	0.900	90		**1/6**	0.167	16 2/3
8/9	0.889	88 8/9		**1/7**	0.143	14 2/7
7/8	0.875	87 1/2		**1/8**	0.125	12 1/2
5/6	0.833	83 1/3		**1/9**	0.111	11 1/9
4/5	0.800	80		**1/10**	0.100	10
3/4	0.750	75		**1/11**	0.0909	9 1/11
2/3	0.667	66 2/3		**1/12**	0.0833	8 1/3
5/8	0.625	62 1/2		**1/15**	0.0667	6 2/3
3/5	0.600	60		**1/16**	0.0625	6 1/4
1/2	0.500	50		**1/20**	0.0500	5
2/5	0.400	40		**1/25**	0.0400	4
3/8	0.375	37 1/2		**1/30**	0.0333	3 1/3
1/3	0.333	33 1/3		**1/32**	0.03125	3 1/8
1/4	0.250	25		**1/40**	0.0250	2 1/2
1/5	0.200	20		**1/50**	0.0200	2

The Only Thing You Need to Remember in Algebra is...

Do the same thing to both sides of the equation!

Example - solve for x

$$3x + 7 = 12$$

add -7 to **both** sides

$$3x + 7 - 7 = 12 - 7$$

$$3x = 5$$

multiple **both** sides by one-third

$$\tfrac{1}{3}\left(3x\right) = 5\left(\tfrac{1}{3}\right)$$

$$x = \frac{5}{3}$$

Now you try it.

$$\frac{x}{7} - 12 = -8$$

See the next section for the solution.

Algebra: Answer

Do the same thing to both sides of the equation!

$$\frac{x}{7} - 12 = -8$$

Add 12 to both sides.

$$\frac{x}{7} - 12 + 12 = -8 + 12$$
$$\frac{x}{7} = 4$$

Now multiply both side by 7.

$$(7)\frac{x}{7} = 4(7)$$
$$x \ = 28$$

Algebra: Three Variables

You will find that we very often use equations or mathematical definitions in chemistry that contain three variables. We will give you some important examples of these in the forthcoming pages, but for now, we want to address the algebra of manipulating these equations.

And remember: Do the same thing to both sides of the equation!

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If you have:

$$x = yz$$

then

$$\left(\frac{1}{y}\right)x = yz\left(\frac{1}{y}\right)$$
$$\frac{x}{y} = z$$

Or

$$z = \frac{x}{y}$$

Using the same methods, you can show that:

$$y = \frac{x}{z}$$

And so, we can express the original equation in 3 different, but equivalent ways:

$$z = \frac{x}{y}\ , \quad x = yz\ , \quad y = \frac{x}{z}$$

These are all different versions of the same equation.

Algebra: Important Definitions

As we noted on the previous page, many important physical quantities are defined in terms of two other variables, giving a total of three variables. Three important definitions are those for density, concentration and pressure, and, in the boxes below, we show how to apply the basic rule of algebra (Do the same thing to both sides of the equation!) to the equations that define them.

Density

We use the Greek letter ρ(pronounced row) for density.

$$\rho = \frac{m}{V}$$

Where "*m*" equals mass and "*V*" equals volume

It again follows that:

$$m = \rho V$$

$$V = \frac{m}{\rho}$$

Density is a ratio. It tells us how much mass is in each volume (choose your units) of a substance.

Concentration

In chemistry, as you will see, we denote concentration by "*M*"

$$M = \frac{n}{V}$$

$$n = MV$$

$$V = \frac{n}{M}$$

Where "*V*" equals the volume in liters and

"*n*" equals the number of moles.

The letter "*M*" stands for Molarity and we will soon have a great deal more to say about it. Although there are many different ways of expressing concentration, Molarity is by far the the most important of them in use in chemistry.

Pressure

$$P = \frac{F}{A}$$

$$F = PA$$

$$A = \frac{F}{P}$$

Where "*F*" equals the force and "*A*" equals the area.

Remember - you are supposed to be learning algebra. So, please be sure that you can carry out these manipulations and that they make sense to

you. Don't be satisfied by mere memorization of these equations.

Algebra: Checking the Units

As we noted on the previous page, many important physical quantities are defined in terms of two other variables, giving a total of three variables. Three important definitions are those for density, concentration and pressure, and, in the boxes below, we show how to apply the basic rule of algebra (Do the same thing to both sides of the equation!) to the equations that define them.

Checking Equations

Did I remember the formula for density correctly?

Here is what I think it is:

$$\rho = \frac{V}{m}$$

Let's see; if I use this formula, my answer will be in:

$$\left(\frac{m^3}{kg}\right)$$

This can't be right.

It must be

$$\rho = \frac{m}{V} \qquad \left(\frac{kg}{m^3}\right)$$

Yes! That is it.

This is how you use dimensional analysis to check on algebraic equations.

Verifying Algebra

The density of lead is 13kg/m. What volume of lead weighs 6 kilograms?

Ok....let's see.

$$\rho = \frac{m}{V}$$

So...

$$V = \frac{m}{\rho} = \frac{6\ \text{kg}}{13\ {}^{\text{kg}}/_{\text{m}^3}}$$

This works because kilograms cancels out and cubic meters (a volume) turns up in the numerator.

If we had made a mistake...

$$V = \frac{\rho}{m} = \frac{13\ {}^{\text{kg}}/_{\text{m}^3}}{6\ \text{kg}}$$

Cubic meters would have shown up in the denominator (wrong units for a volume) and we would have known our answer was wrong.

SI Units

We have to agree on the units we will use.

1. Centimeters?

2. Inches?

3. Furlongs?:

People **have** agreed on the MKS system.
Meters are used to measure length.
Kilograms are used to measure mass.
Seconds are used to measure time.

If we use these all the time, we will always get dimensionally consistent answers. You should have encountered these units in previous science and chemistry courses.

Guess what? We won't always use them. We'll use them most of the time, but science is a human enterprise, and people sometimes resist standardization.

Greek Prefixes

We generally use prefixes derived from the Greek language to express very large or very small numbers:

Big			**Little**	
Kilo	10^{3}		Milli	10^{-3}
Mega	10^{6}		Micro	10^{-6}
Giga	10^{9}		Nano	10^{-9}
Tera	10^{12}		Pico	10^{-12}
			Femto	10^{-15}

Why do we use these? Because sometimes numbers get too big or too small.

For example, light travels 1 foot in 0.000000001 seconds.

How about, light travels 1 foot in one nanosecond?

Much easier don't you think?

Unit Conversions Ex1

Every conversion between different units, involves a ratio between two numbers. This ratio will always give you 2 possible conversion factors. You must use dimensional analysis to figure out which conversion factor to use.

Example:

How many feet are there in 5 yards?

We know there are three feet in one yard. This can be expressed as:

$$\frac{3 \text{ feet}}{1 \text{ yard}} \qquad \text{OR} \qquad \frac{1 \text{ yard}}{3 \text{ feet}}$$

Note the two ratios.

Which to use?

$$5 \text{ yards} \times \frac{1 \text{ yard}}{3 \text{ feet}} = ?????$$

$$5 \text{ yards} \times \frac{3 \text{ feet}}{1 \text{ yard}} = 15 \text{ feet}$$

The second expression works because the yards cancel out and we are left with feet. That's all there is to dimensional analysis.

Turn to the next page to see another example.

Unit Conversions Ex2

If I am Traveling at 25 m/sec, How Fast is That in Miles Per Hour?

Assume that there are 0.6 m in 1km.

$$\frac{25\text{m}}{\text{sec}} \times \frac{3600\text{sec}}{\text{hr}} \times \frac{1\text{km}}{1000\text{m}} \times \frac{0.6\text{mi}}{\text{km}}$$
$$= 54^{\text{miles}}/_{\text{hour}}$$

Here are two for you to try.

1. How many square centimeters in 3 square feet?

2. How many feet does light travel in 1 nanosecond?

C=3.0 x 10^8 m/sec
1 nanosecond = 10^{-9} seconds

Please see the next section for the answers.

Unit Conversions Answers

1. How many sq cm in 3 sq ft?

$$\frac{3\text{ft}^2}{1}\left(\frac{12\text{ in}}{1\text{ ft}}\right)\left(\frac{12\text{ in}}{1\text{ ft}}\right)\left(\frac{2.54\text{ cm}}{1\text{ in}}\right)\left(\frac{2.54\text{ cm}}{1\text{ in}}\right)$$

$$= 2787\text{ cm}^2$$

Notice how we needed feet squared in the denominator to cancel out feet squared in numerator.

2. How fast does light travel in feet/nanosecond?

$$\left(\frac{3\times 10^8\text{m}}{\text{s}}\right)\left(\frac{10^{-9}\text{s}}{\text{ns}}\right)\left(\frac{100\text{ cm}}{\text{m}}\right)\left(\frac{1\text{ in}}{2.54\text{ cm}}\right)\left(\frac{1\text{ ft}}{12\text{ in}}\right)$$

$$= 0.98\ {}^{\text{feet}}/_{\text{ns}}$$